18 A child of mass 35 kg is standing on a trampoline. At equilibrium the surface of the trampoline is displaced vertically by 22 cm from the unloaded position.

(a) Show that the force constant of the trampoline is about 1600 N m^{-1}.

(2)

At the equilibrium position $F_{net} = 0$ => $W = F_{tr}$ => $m \times g = F_{tr}$ => $F_{tr} = 35 \times 9.81$ => $F_{tr} = 343.35$ N

For the force from the trampoline $F_{tr} = k \times \Delta L$. At the equilibrium position $F_{tr} = k \times \Delta L_0$ => $343.35 = k \times 0.22$ =>

$k = 1560.6818 = 1600$ N.m^{-1}

(b) The child bounces up and down, always staying in contact with the trampoline. The motion is simple harmonic.

(i) Calculate the child's frequency of oscillation.

(3)

For the SHM $F_{net} = - m\omega^2 x$ => $F_{tr} - W = - m\omega^2 x$ => $k \times (\Delta L_0 - x) - m \times g - = - m\omega^2 x$ => $k = m \times \omega^2$

Thus ω = sqrt(k / m) = 6.676 rad.s^{-1}. As $\omega = 2\pi f$ => f = 1.0625 Hz = 1.1 Hz

Frequency of oscillation = f = 1.0625 Hz = 1.1 Hz

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(ii) The height of each bounce above the equilibrium position is 21 cm.

Calculate the maximum speed of the child and identify the position at which she has this speed.

(3)

$v_{max} = \omega x A \Rightarrow v_{max} = 2\pi \times f \times A = 2\pi \times 1.1 \times 0.21 = 1.4514 = 1.4\ m.s^{-1}$

Maximum speed of child = $v_{max} = 1.4\ m.s^{-1}$

Position = Equilibrium position

(c) (i) The child bends her knees and pushes against the surface of the trampoline at each bounce. Her amplitude of oscillation gradually increases.

Name this effect and explain why there is an increase in amplitude.

(3)

Name of effect Resonance.

Explanation When the child pushes against the surface of the trampoline, through the work of her force she transfers energy to the trampoline periodically. Thus we have a forced oscillation. As the amplitude of oscillation increases we conclude that the frequency of the force is close or equal to the natural frequency of the system. Thus we have an optimum transfer of energy. The phenomenon is called resonance.

*(ii) As her amplitude of oscillation increases she starts to lose contact with the surface of the trampoline.

Explain why the motion can no longer be described as simple harmonic.

(3)

A motion is characterised as SHM when the net (resultant) force on the oscillator is proportional and opposite to the displacement of the oscillator from its equilibrium position. When the child loses contact with the trampoline the only force acting on her is her weight. Thus the condition for the SHM is violated.

(Total for Question 18 = 14 marks)

TOTAL FOR SECTION B = 70 MARKS

TOTAL FOR PAPER = 80 MARKS

List of data, formulae and relationships

Acceleration of free fall	$g = 9.81\ \text{m s}^{-2}$	(close to Earth's surface)
Boltzmann constant	$k = 1.38 \times 10^{-23}\ \text{J K}^{-1}$	
Coulomb's law constant	$k = 1/4\pi\varepsilon_0$	
	$= 8.99 \times 10^{9}\ \text{N m}^2\ \text{C}^{-2}$	
Electron charge	$e = -1.60 \times 10^{-19}\ \text{C}$	
Electron mass	$m_e = 9.11 \times 10^{-31}\ \text{kg}$	
Electronvolt	$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$	
Gravitational constant	$G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$	
Gravitational field strength	$g = 9.81\ \text{N kg}^{-1}$	(close to Earth's surface)
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{F m}^{-1}$	
Planck constant	$h = 6.63 \times 10^{-34}\ \text{J s}$	
Proton mass	$m_p = 1.67 \times 10^{-27}\ \text{kg}$	
Speed of light in a vacuum	$c = 3.00 \times 10^{8}\ \text{m s}^{-1}$	
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}$	
Unified atomic mass unit	$u = 1.66 \times 10^{-27}\ \text{kg}$	

Unit 1

Mechanics

Kinematic equations of motion	$v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$
Forces	$\Sigma F = ma$ $g = F/m$ $W = mg$
Work and energy	$\Delta W = F\Delta s$ $E_k = \frac{1}{2}mv^2$ $\Delta E_{grav} = mg\Delta h$

Materials

Stokes' law	$F = 6\pi\eta rv$
Hooke's law	$F = k\Delta x$
Density	$\rho = m/V$
Pressure	$p = F/A$
Young modulus	$E = \sigma/\varepsilon$ where Stress $\sigma = F/A$ Strain $\varepsilon = \Delta x/x$
Elastic strain energy	$E_{el} = \frac{1}{2}F\Delta x$

Unit 5

Energy and matter

Heating	$\Delta E = mc\Delta\theta$
Molecular kinetic theory	$\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$
Ideal gas equation	$pV = NkT$

Nuclear Physics

Radioactive decay	$\mathrm{d}N/\mathrm{d}t = -\lambda N$
	$\lambda = \ln 2/t_{\frac{1}{2}}$
	$N = N_0 \mathrm{e}^{-\lambda t}$

Mechanics

Simple harmonic motion	$a = -\omega^2 x$ $a = -A\omega^2 \cos \omega t$ $v = -A\omega \sin \omega t$ $x = A \cos \omega t$ $T = 1/f = 2\pi/\omega$
Gravitational force	$F = Gm_1m_2/r^2$

Observing the universe

Radiant energy flux	$F = L/4\pi d^2$
Stefan-Boltzmann law	$L = \sigma T^4 A$ $L = 4\pi r^2 \sigma T^4$
Wien's Law	$\lambda_{max} T = 2.898 \times 10^{-3}$ m K
Redshift of electromagnetic radiation	$z = \Delta\lambda/\lambda \approx \Delta f/f \approx v/c$
Cosmological expansion	$v = H_0 d$